Phenomenology of Majorana zero modes in full-shell hybrid nanowires

Carlos Payá

Instituto de Ciencia de Materiales de Madrid (ICMM), CSIC

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Outline

The Kitaev chain

- Chain of N spin-less fermions (p-wave superconductivity):

$$H = -\mu \sum_{j=1}^{N} \left(c_j^\dagger c_j - \frac{1}{2} \right) + \sum_{j=1}^{N-1} \left[-t \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) + \Delta \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger \right) \right]$$

(a)

R. Aguado 2017, *Rivista del Nuovo Cimento*.
E. Prada *et al.* 2020, *Nature Reviews Physics*.
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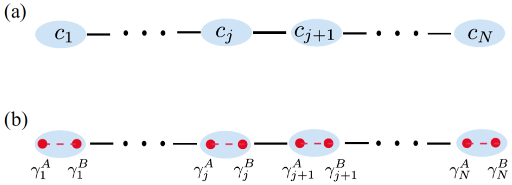
- Majorana representation:

$$c_j = \tfrac{1}{2} \left(\gamma_j^A + i\gamma_j^B \right), \quad c_j^\dagger = \tfrac{1}{2} \left(\gamma_j^A - i\gamma_j^B \right)$$



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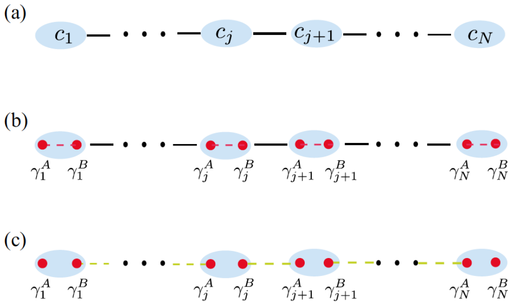
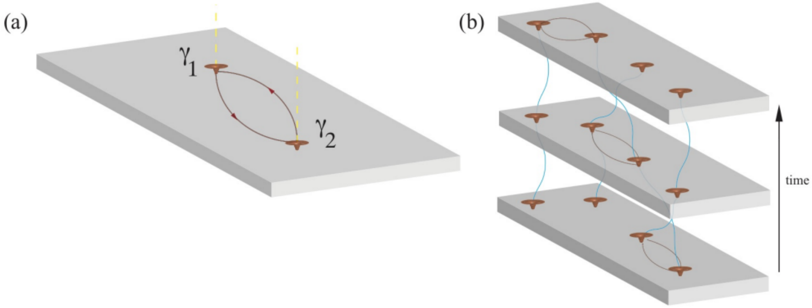
- Hamiltonian in terms of Majorana operators:

$$H = -\frac{i\mu}{2} \sum_{j=1}^{N} \gamma_j^A \gamma_j^B + \frac{i}{2} \sum_{j=1}^{N-1} \left[(\Delta + t) \gamma_j^B \gamma_{j+1}^A + (\Delta - t) \gamma_j^A \gamma_{j+1}^B \right]$$



Majoranas for qubits



- MZM are non-Abelian anyons.
- Gap closing/reopening $\Rightarrow$ topological protection.

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We need a $p-$wave superconductor!

- The superconducting pairing term in the Kitaev chain is spinless: $\Delta\left(c_j c_{j+1} + c^\dagger_{j+1} c^\dagger_j\right)$.

L. Fu and C. L. Kane 2008, *Phys. Rev. Lett.*
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- Lutchyn and Oreg: proximitize semiconductors with strong spin-orbit coupling.

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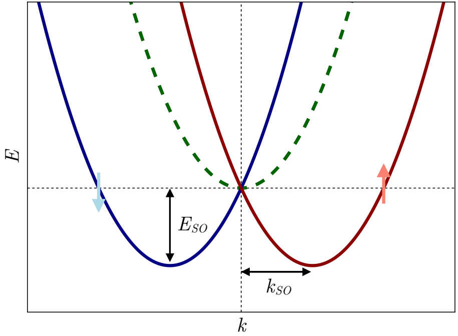
Rashba, Zeeman and helical bands



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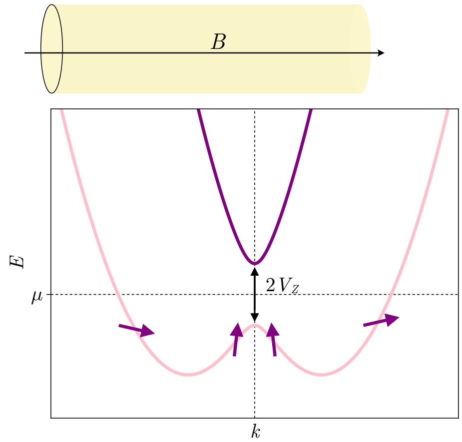

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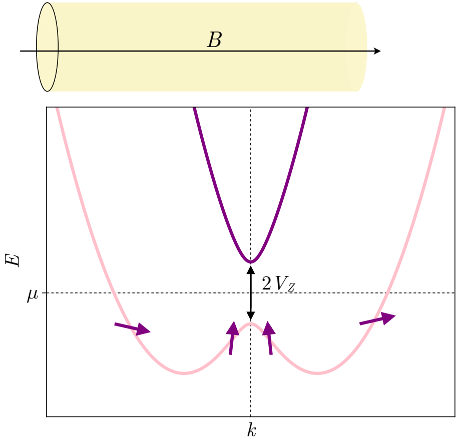

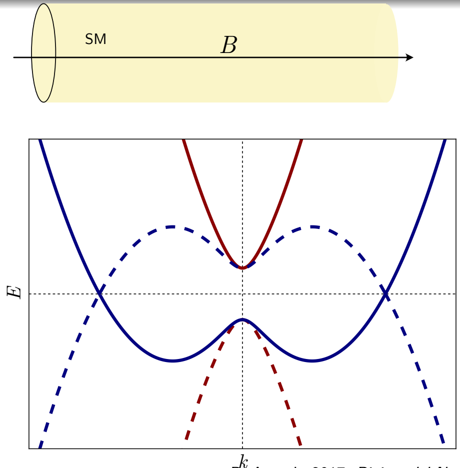

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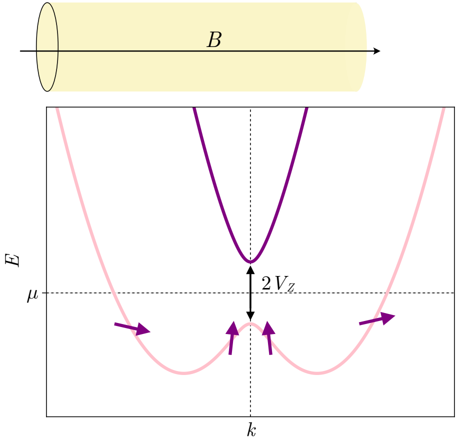

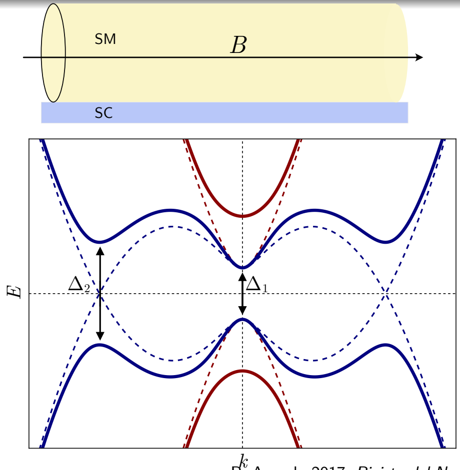

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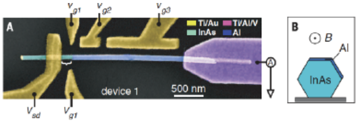
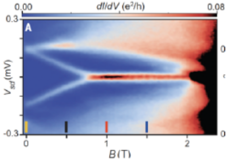
Searching for Majoranas

- Strong experimental interest.



Zero-bias anomalies in tunneling spectroscopy experiments

Mourik 2012, Albrecht 2016, Deng 2016



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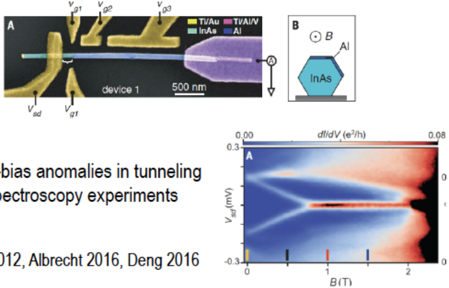
Searching for Majoranas

- Strong experimental interest.
- Zero-bias anomalies detected with non-topological explanations.



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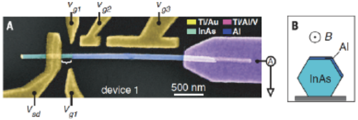
Searching for Majoranas

- Drawbacks:



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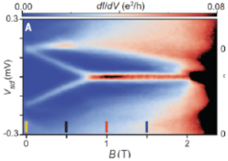

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Searching for Majoranas

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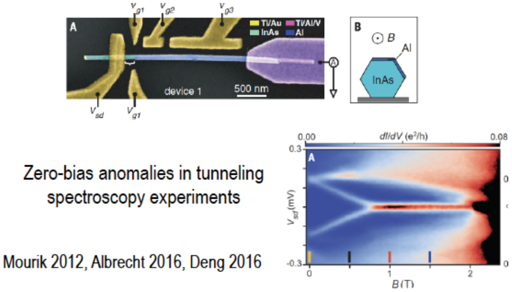

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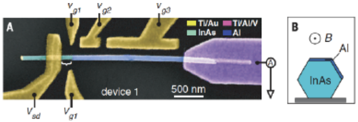
Searching for Majoranas

- Drawbacks:
  - Multimode effects.
  - Electrostatic environment.



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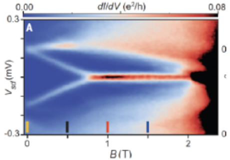

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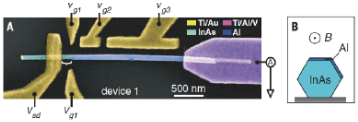

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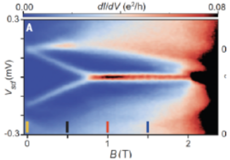

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Searching for Majoranas

- Drawbacks:
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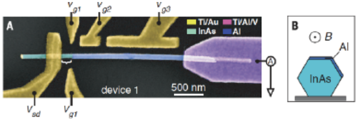
  - High magnetic fields.



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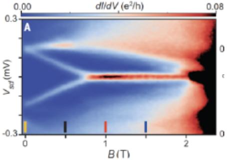

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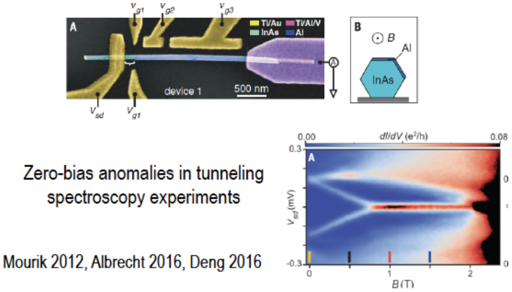

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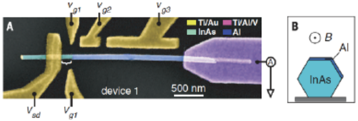

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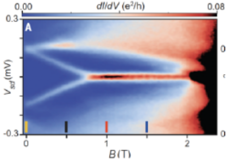

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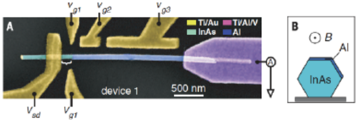

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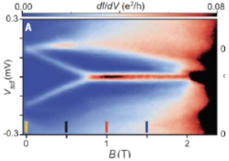

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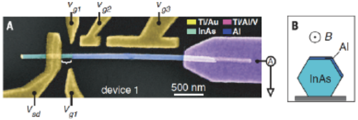

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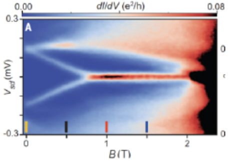

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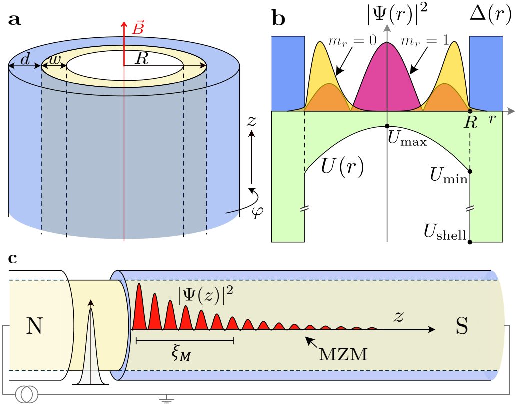
The full-shell nanowire



- Key points:

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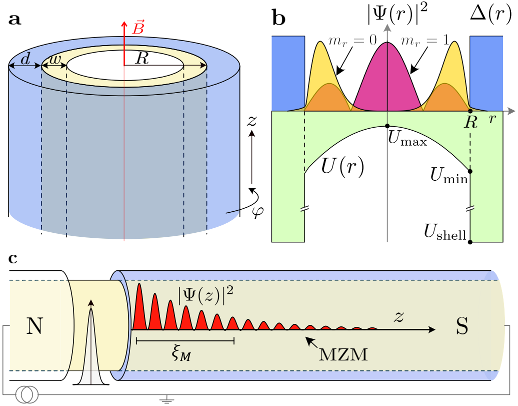

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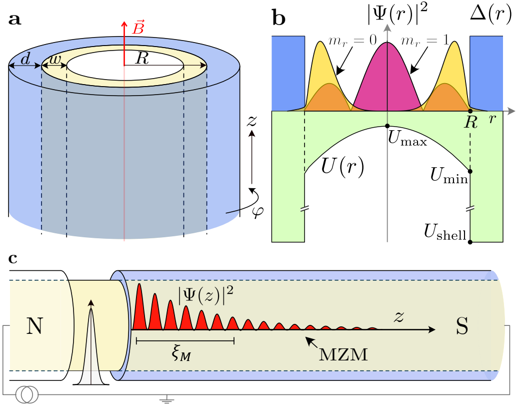

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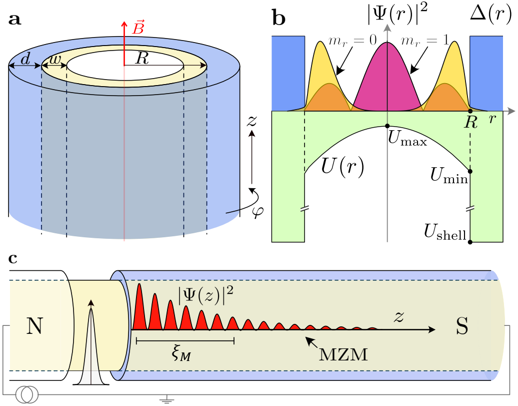

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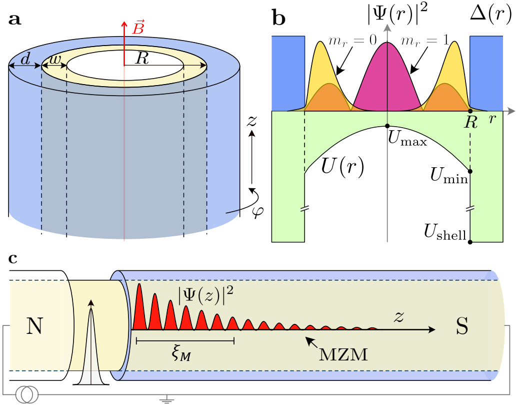

- Key points:
  - Cylindrical symmetry
  - Topological transition driven by orbital effect **not Zeeman**
  - Needs lower magnetic fields.
  - Only one angular mode can be topological.

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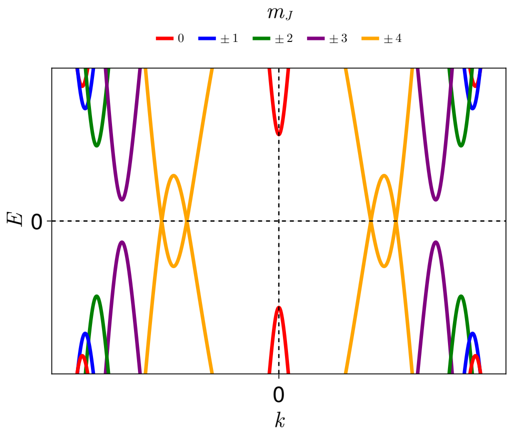
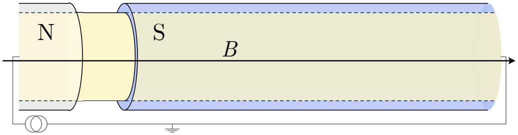
The CdGM analog states





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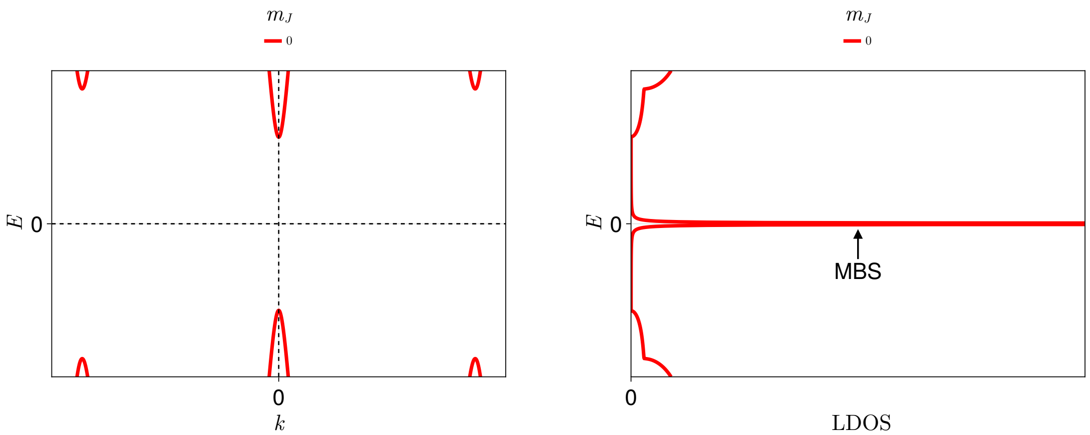

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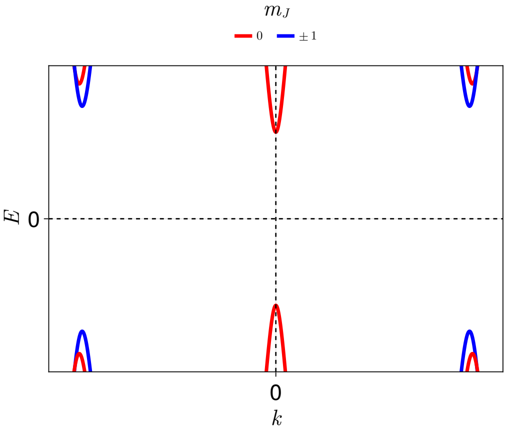

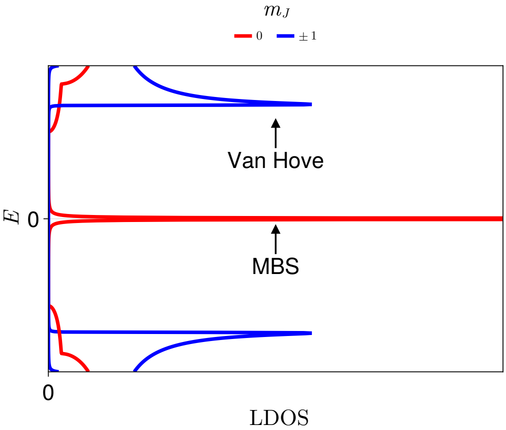

C. Payá *et al.* 2024, *Phys. Rev. B.*
P. San-Jose *et al.* 2023, *Phys. Rev. B.*

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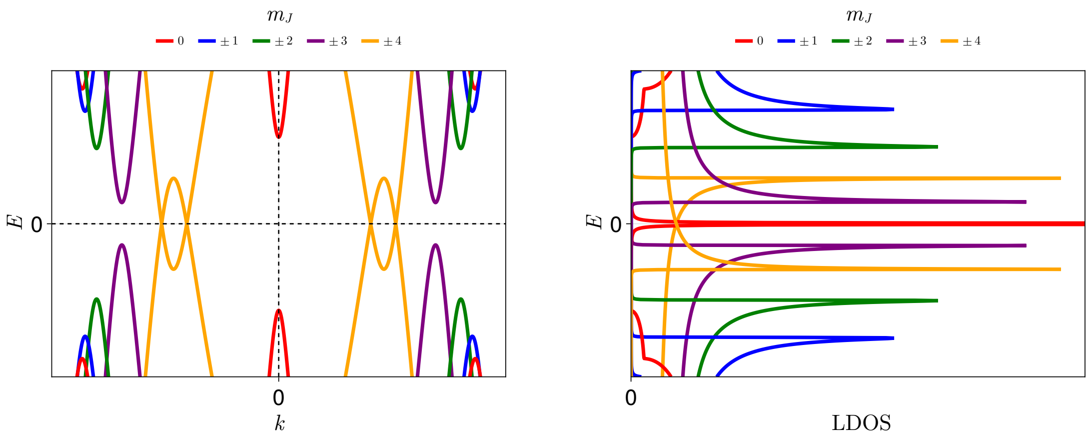
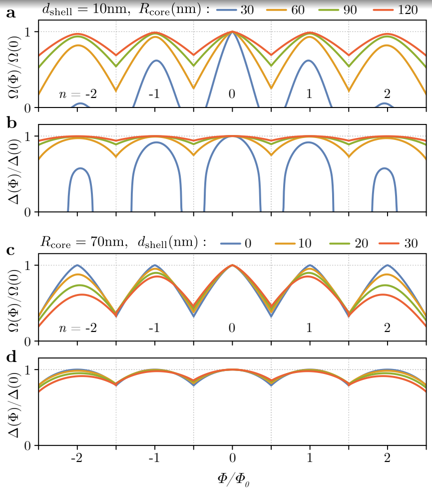


C. Payá *et al.* 2024, *Phys. Rev. B.*
P. San-Jose *et al.* 2023, *Phys. Rev. B.*

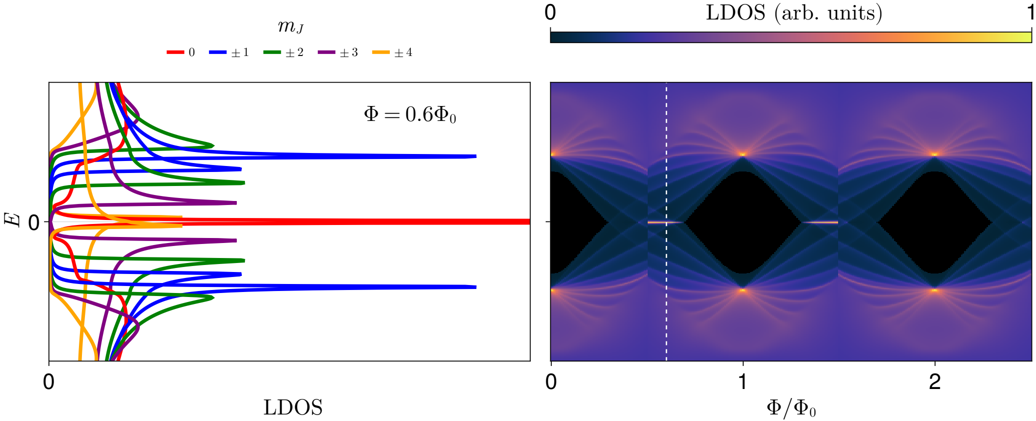
Digression: the Little-Parks effect



- Cylinder $\Leftrightarrow$ vortex.
- Too thin for full Meissner.
- Quantized winding of the order parameter: $\Delta = |\Delta| e^{in\varphi}$.
- $n \in \mathbb{Z}$ and jumps every flux quantum Φ_0.
- Quasi-quantization of flux $\Rightarrow$ pairing presents LP lobes.
- Depends on R, SC thickness d and ξ_d, the SC coherence length.

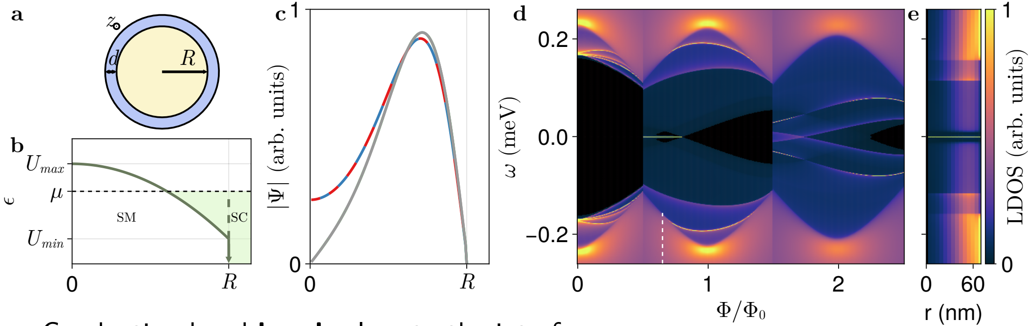
W. A. Little and R. D. Parks 1962, *Phys. Rev. Lett.*
R. D. Parks and W. A. Little 1964, *Phys. Rev.*

LDOS vs. flux



C. Payá *et al.* 2024, *Phys. Rev. B.*
P. San-Jose *et al.* 2023, *Phys. Rev. B.*

Pushing the WF to the interface



- Conduction band **bends** close to the interface.

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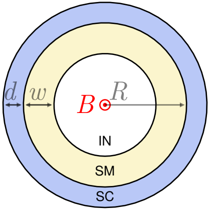
Protected islands in the tubular-core



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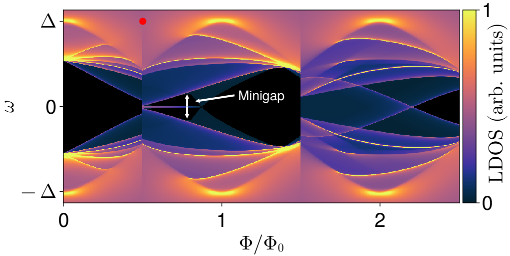

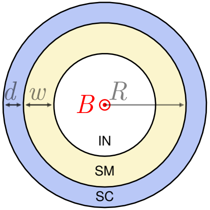

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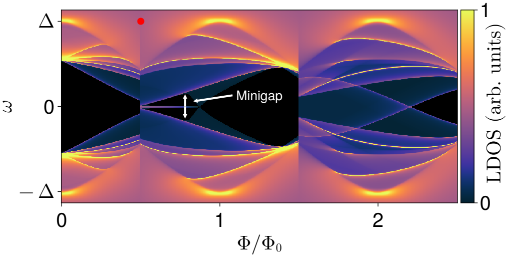
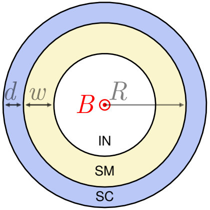


- We need to push the charge to the interface.



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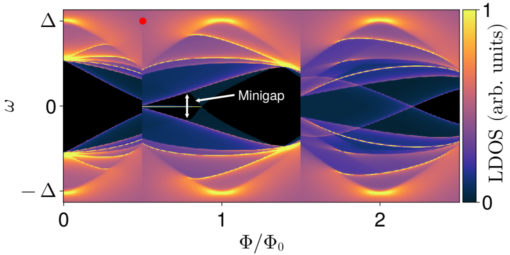
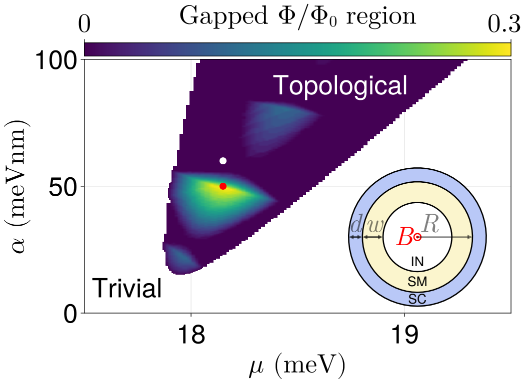


- We need to push the charge to the interface.
- Topologically protected islands appear-



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Summary

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    1. Majorana zero modes coexist with CdGM analog states.

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Take home message

Majorana physics of full-shell nanowires is very rich. For pristine configurations, the tubular-core model is the optimal candidate in comparison to the solid-core geometry.

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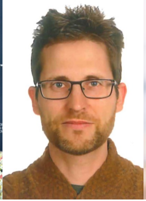
People involved

Project Leader

Elsa Prada



Pablo San José

Ramón Aguado

Theory:
Samuel D. Escribano (Weizmann Institute)
Andrea Vezzosi (U. of Modena, now U. of Lausanne)
Fernando Peñaranda (DIPC)

Experimentalists:
Saulius Vaitiekėnas (Niels Bohr Institute)
Charles M. Marcus (NBI, now U. of Washington)

Ongoing experiments:
Jesper Nygård (Niels Bohr Institute)

MINISTERIO DE CIENCIA E INNOVACIÓN
UNIÓN EUROPEA
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AGENCIA ESTATAL DE INVESTIGACIÓN

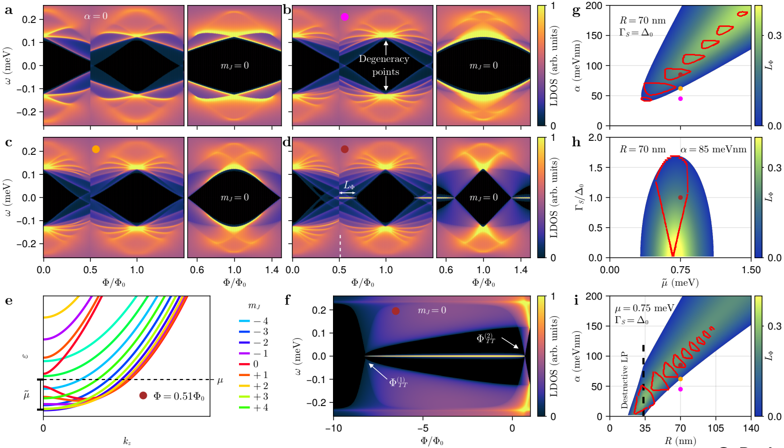
Phenomenology of Majorana zero modes in full-shell hybrid nanowires

Carlos Payá

Instituto de Ciencia de Materiales de Madrid (ICMM), CSIC

July 9, 2024

Hollow-core results



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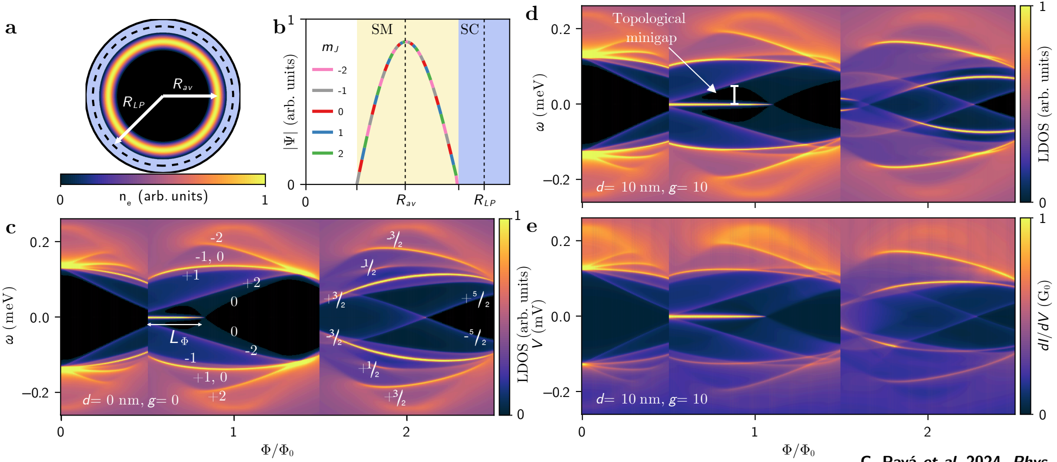
Modified hollow-core results



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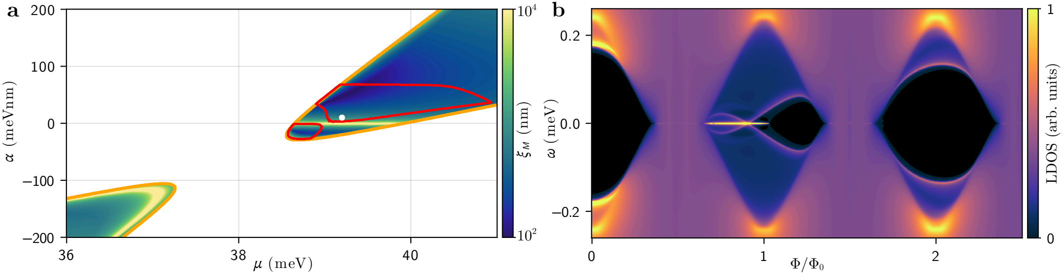
Destructive Litle-Parks



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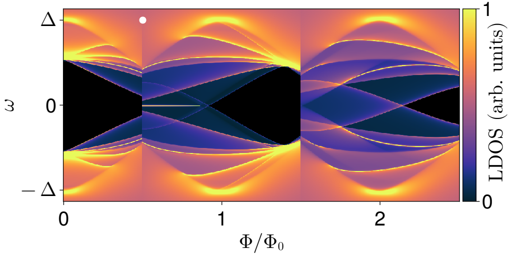
The tubular-core model



- Adding a width to the semiconductor.

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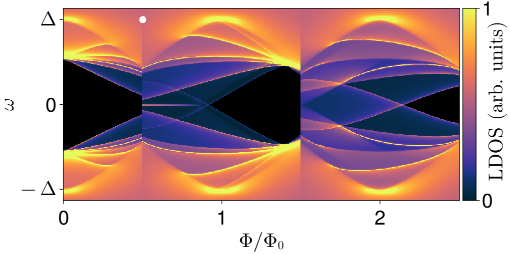
The tubular-core model



- Adding a width to the semiconductor.
- Most common scenario: CdGMs fill the MZM minigap.
- No topological protection

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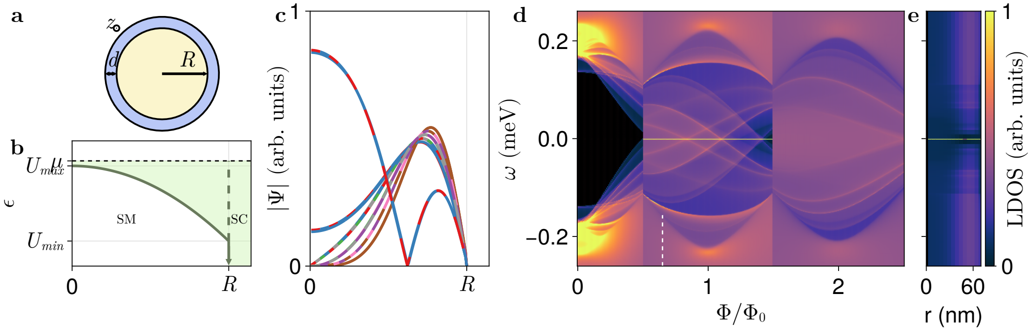
Second radial mode: protection lost



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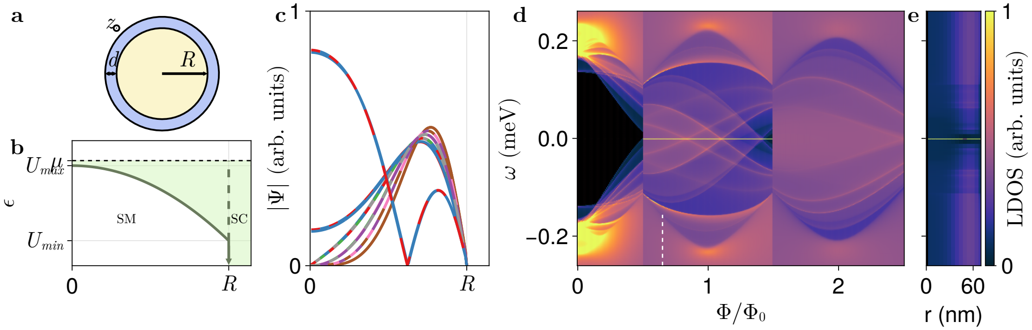

- When the second radial mode is occupied, the ZEP expands over the full lobe, but CdGMs cover it.

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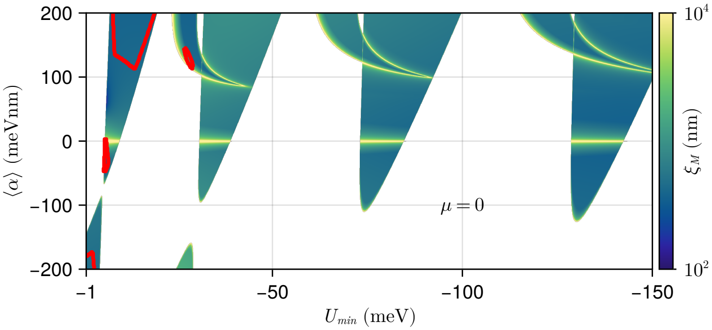
Band-bending: not enough islands



- Notice axis are mean α and U_{min}, the minimum of the dome-profile.
- One wedge per radial mode. No islands outside the first radial mode.

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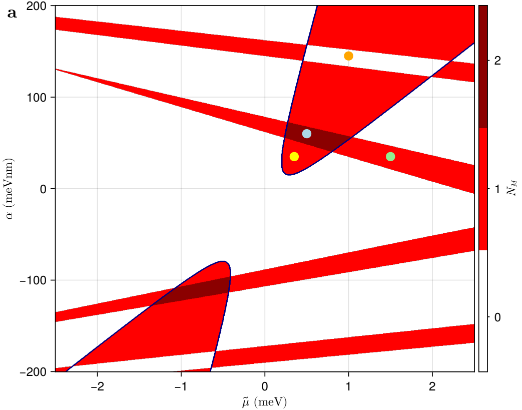
Hexagonal wave-function



- New red stripes. Hexagon has $\ell = 6$.
- Upper stripe: $m_J = 0$ mixes with $m_J = \pm 6$.
- Lower stripe: $m_J = 3$ mixes with $m_J = -3$.
- The MZM coming from $m_J = \pm 3$ **cannot** interact with $m_J = 0 \Rightarrow$ they overlap.
- The $m_J = \pm 6$ MZM annihilates the $m_J = 0$ MZM.

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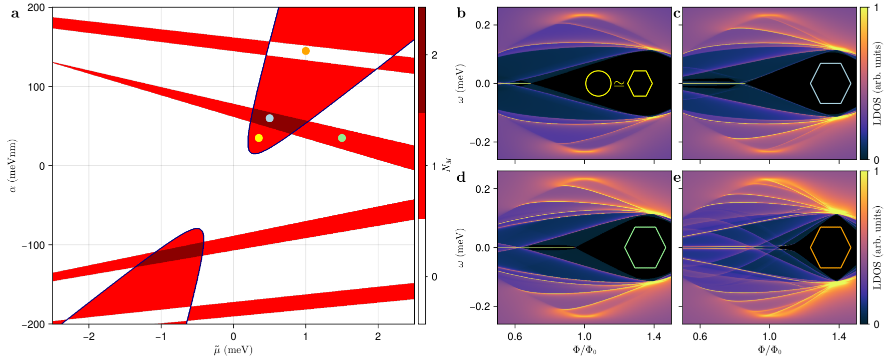
Hexagonal wave-function



- Except for the new topological stripes and a region where the MZM splits, the system is equivalent to the cylinder.

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